The Height of a Giraffe [*]

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Abstract

A minor modification of the arguments of Press and Lightman leads to an estimate of the height of the tallest running, breathing organism on a habitable planet as the Bohr radius multiplied by the three-tenths power of the ratio of the electrical to gravitational forces between two protons (rather than the one-quarter power that Press got for the largest animal that would not break in falling over, after making an assumption of unreasonable brittleness). My new estimate gives a height of about 3.6 meters rather than Press's original estimate of about 2.6 cm. It also implies that the number of atoms in the tallest runner is very roughly of the order of the nine-tenths power of the ratio of the electrical to gravitational forces between two protons, which is about 3×10^{32}.

Introduction

Press [1] and Press and Lightman [2] have given estimates of our size as being of the order of $(e/m_p)^{0.5}a_0$, which is about 0.0558 meters or nearly 6 cm, using Planck units $\hbar = c = G = 4\pi\epsilon_0 = k = 1$, with k being Boltzmann's constant. (Press [1] actually included some numerical factors that reduced his estimate to about 2.6 cm.) Here I give a slightly modified argument that gives a size estimate of the order of $(e/m_p)^{0.6}a_0$, which leads to a more accurate value of 3.56 meters.

Press [1] uses three "requirements": "(i) We are made of complicated molecules; (ii) we breathe an evolved planetary atmosphere; (iii) we are about as big as we can be without breaking." In more detail, Press states the following:

"Let us assume only that man [the term used generically to include male as well as female] satisfies three properties: (i) he is made of complicated molecules; (ii) he requires an atmosphere which is not (primordial, cosmological) hydrogen and helium; and (iii) he is as large as possible, to carry his huge brain, but he is liable to stumble and fall; and in so doing he should not break. These three properties do not differentiate between a man and, say, an elephant of size L_E; however $L_E \approx L_H$ to the accuracy of our calculation, and we should not expect to distinguish elephants from men by dimensional arguments."

Here I shall accept Press's assumptions (i) and (ii) but instead of using his (iii), I shall use the assumption that the organism in question is the tallest organism that can run without overheating. Strictly speaking, this would better give the height of a giraffe rather than that of a man, and coincidentally the ignored numerical factors conspire to make that true, but as Press mentioned in comparing a man and an elephant, my estimate for the height of a giraffe is also a rough estimate of our height as another species of the largest running animals on earth.

In this argument, it is convenient to define three dimensionless quantities, which I shall give here first in Planck units and then in in conventional units (if the expressions differ): (a) the fine-structure constant,

$$\alpha \equiv e^2 \equiv \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.036\,00} \approx 0.007\,297\,352\,6, \tag{1}$$

the ratio of the electron mass to the proton mass,

$$\beta \equiv \frac{m_e}{m_p} \approx \frac{1}{1\,836.152\,763} \approx 0.000\,544\,617\,021\,6, \tag{2}$$

and the ratio of the electrical repulsion to the gravitational attraction between two protons,

$$\gamma \equiv \frac{e^2}{m_p^2} \equiv \frac{e^2}{4\pi\epsilon_0 G m_p^2} \approx 1.236 \times 10^{36}. \tag{3}$$

Then, for example, in terms of the Planck length $L_P \equiv \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}$ m and the Planck mass $M_P \equiv \sqrt{\hbar c/G} \approx 2.176 \times 10^{-8}$ kg $\approx 1.311 \times 10^{19}$ amu, which

I am setting to unity when I use Planck units, one can write the Bohr radius as

$$a_0 \equiv m_e^{-1} e^{-2} = \alpha^{-3/2} \beta^{-1} \gamma^{1/2} L_P \approx 0.529\,177\,21 \times 10^{-10}\,\text{m}. \tag{4}$$

One can also define a crude estimate for a typical stellar mass [2] as

$$M_s \equiv m_p^{-2} = \alpha^{-1} \gamma M_P \approx 3.685 \times 10^{30} \text{kg} \approx 1.853\,M_\odot, \tag{5}$$

where $M_\odot \approx 1.988 \times 10^{30}$ kg is the mass of the sun. If the mass M_s were a black hole, the corresponding length scale (half the Schwarzschild radius) would be

$$L_s \equiv m_p^{-2} = \alpha^{-1} \gamma L_P \approx 2\,737\,\text{m}. \tag{6}$$

Below it is convenient to write estimates of the mass M and radius R of a habitable planet in terms of M_s and L_s and the dimensionless ratios α, β, and γ, and to write animal sizes in terms of a_0 and these same dimensionless ratios.

1 Press and Press-Lightman Estimates

Requirement (i) in Press [1], that we are made of complicated molecules, leads to the requirement that the environmental temperature T have an energy equivalent kT that is less than the binding energy of those molecules. On the other hand, the mobility assumed by (iii) implies that kT is not enormously less than the binding energies, or else the internal energy processes would occur exponentially slowly. These requirements lead to an environmental temperature T whose equivalent energy kT is a small fraction, say ϵ, of the Rydberg energy,

$$\text{Ry} \equiv \frac{1}{2} m_e e^4 \sim m_e \alpha^2 = \alpha^{5/2} \beta \gamma^{-1/2} M_P c^2, \tag{7}$$

dropping the factor of $\frac{1}{2}$ in the final two expressions. Press [1] says that a reasonable value for the small fraction ϵ is 0.003; Press and Lightman [2] take the small fraction to be $\varepsilon(m_e/m_p)^{1/2}$ with $\varepsilon \sim 0.1$, which would give $\epsilon = \varepsilon \beta^{1/2} \approx 0.0023$.

Actually, Press and Lightman use the same ϵ symbol as Press does, even though they are different quantities, but here I shall distinguish the Press-Lightman one by writing it as ε. For simplicity in the analysis below, I shall drop numerical factors of the rough order of unity and set the Press-Lightman ε to unity to make the Press $\epsilon = \beta^{1/2} = (m_e/m_p)^{1/2} \approx 0.0233$. The fact that this is almost 8 times larger than the value $\epsilon \sim 0.003$ advocated in Press [1] and is 10 times larger than the value advocated in Press and Lightman [2] is often partially compensated by my neglect of other numerical factors, such as the ratio of a_0^3 to the space taken up by an atom, so when I neglect these other factors, I find that it usually doesn't help much, if any, to include the ε factor.

At the end of the Press-Lightman presentation [2], it is recorded that Rudolf Peierls objected to $\varepsilon(m_e/m_p)^{1/2}$ Ry as an estimate of molecular binding energies, asking whether $(m_e/m_p)^{1/2}$ Ry instead represents "an estimate of the zero-point

energy of vibration," which "would seem a considerable underestimate, since the vibrational energy usually amounts to many vibrational quanta." Press concurred, "Yes, the factor of $(m_e/m_p)^{\frac{1}{2}}$ does strictly give the characteristic energy-level *spacing* of molecules, rather than their full binding energy. Numerically, however, it does also give the correct (rough) factor by which molecular bindings are smaller than typical atomic ones. One may wish to consider the factor a *mnemonic* for this dimensionless ratio of binding energies (which derives from a theory of chemistry) rather than an accurate physical 'theory' of that ratio."

In our part of the universe, where $\beta^{1/2} = (m_e/m_p)^{1/2} \approx 0.0233$, it is thus numerically okay to use this as a rough estimate for the "factor by which molecular bindings are smaller than typical atomic ones," and in this paper I shall do that. However, for other possible parts of our universe or multiverse that have electrons and protons but with potentially greatly different mass ratios, it may no longer be even roughly valid to use $(m_e/m_p)^{1/2}$ for the ratio of biological molecular binding energies to the Rydberg in that part of the universe. Therefore, quantities below that depend upon $\beta \equiv m_e/m_p \approx 1/1\,836.152\,673$ should be interpreted cautiously for other parts of the universe where this small ratio β is significantly different. However, in my estimate below for the height of the largest running organism, β enters with only the one-twentieth power, so one can effectively drop the β dependence (assuming that it is not too many orders of magnitude away from unity), as I do to get the simple formula for the giraffe height as a Bohr radius multiplied by the three-tenths power of the ratio of the electrical to gravitational forces between protons.

If one does set $kT = \beta^{1/2}m_e e^4 = \alpha^2(m_e/m_p)^{1/2}m_e c^2$, where the last expression reverts from Planck units to ordinary units by inserting c^2, one gets that $T \approx 7\,369 K$, which of course is far hotter than the surface of the earth. One would get a much better result by inserting the factor of $1/2$ for the Rydberg and a factor of $\varepsilon = 0.1$, which would give $T \approx 368 K \approx 95 C$, which would still be unbearably hot for humans (nearly boiling) but which would be within 30% of a typical earth surface value. However, since it is both *ad hoc* and difficult to give good estimates of all the numerical factors such as ε, here I shall ignore all of them and just proceed with

$$kT \sim m_e^{3/2}m_p^{-1/2}e^4 = \alpha^2\beta^{1/2}m_e c^2 \approx 0.636\,\text{eV}, \tag{8}$$

where for the penultimate expression on the right I have inserted the factor of c^2 that is unity in Planck units in order to have the correct expression in ordinary units, with the small dimensionless quantities α being the fine-structure constant and β being the ratio of the electron mass to the proton mass.

Requirement (ii) in Press [1], that we breathe an evolved planetary atmosphere, leads to a habitable planet of radius R and mass $M \sim (m_p/a_0^3)R^3$ such that the magnitude of the gravitational binding energy of hydrogen, GMm_p/R, is of the order of $\epsilon kT \sim \beta^{1/2}\text{Ry} \sim (m_e/m_p)^{1/2}m_e e^4$, so that hydrogen and helium can escape from the earth, but not most of the heavier gases. For simplicity I am dropping numerical factors of the order of unity for the volume and density of the earth as well as the Press-Lightman factor ε and the factor of $1/2$ in the Rydberg. This then

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leads to a habitable planetary radius

$$R \sim m_e^{-3/4} m_p^{-5/4} e^{-1} = \alpha^{-3/2} \beta^{-3/4} \gamma L_P = \alpha^{-1/2} \beta^{-3/4} L_s \approx 8\,986 \text{km} \approx 1.409 R_\oplus, \quad (9)$$

$$M \sim m_e^{3/4} m_p^{-11/4} e^3 = \alpha^{1/2} \beta^{3/4} \gamma M_P = \alpha^{3/2} \beta^{3/4} M_s \approx 8.190 \times 10^{24} \text{kg} \approx 1.371 M_\oplus,$$
$$(10)$$

where $R_\oplus \approx 6\,378.140$ km is the equatorial radius of the earth and $M_\oplus \approx 5.972 \times 10^{24}$ kg is the mass of the earth.

Thus although ignoring the factors of $1/2$ and ε made the estimated temperature T come out about 25 times a typical earth surface temperature, for the mass and radius of a habitable planet these factors are mostly canceled by other numerical factors that I am also ignoring, so both M and R are within 37–41% of the values for earth. It is amusing that the resulting crude estimate for the orbital speed of a satellite skimming the planet comes out to be very accurate,

$$v_{\text{satellite}} \sim \alpha \beta^{3/4} \approx 0.2602 \times 10^{-6} c \approx 7.799 \,\text{km/s}, \quad (11)$$

which is just 1.34% smaller than the actual value of $\sqrt{GM_\oplus / R_\oplus} \approx 0.2637 \times 10^{-6} c \approx$ 7.905 km/s for the earth.

For the properties of life on a planet, the main parameters of importance are the temperature T estimated above and the surface gravity that may be estimated as

$$\begin{aligned} g = \frac{GM}{R^2} \sim m_e^{9/4} m_p^{-1/4} e^5 &= \beta^{1/4} m_e^2 e^5 = \alpha^{5/2} \beta^{1/4} m_e^2 = \alpha^{7/2} \beta^{9/4} \gamma^{-1} c^2 L_P^{-1} \\ &= \alpha^{5/2} \beta^{9/4} c^2 L_s^{-1} \approx 6.769 \,\text{m/s}^2 \approx 0.6903 \, g_\oplus, \quad (12) \end{aligned}$$

where $g_\oplus = 9.80665$ m/s^2 is the standard gravitational acceleration on earth.

Having dropped the ε factor of Press and Lightman [2], my β factor is essentially the factor ϵ^2 of Press [1], so one might also write this approximation as $g \sim \epsilon^{1/2} e^5 m_e^2$, which Carter [3] did. He then noted that it is interesting that if one takes ϵ (and not ε) to be independent of β (as it might be in reality, with $\epsilon \sim \beta^{1/2}$ being only a mnemonic formula that works in our part of the universe and not universally), then the approximation for the acceleration of gravity g on a habitable planet depends only on the properties of the electron, and not upon the mass of the proton. However, for agreement within 31% of the acceleration of gravity on earth, it does help to use $\beta^{1/2}$ for ϵ, since dropping the $\beta^{1/4}$ or $\epsilon^{1/2}$ factor altogether gives the more memorable but more crude estimate $g \sim e^5 m_e^2 \approx 44.31$ m/s$^2 \approx 4.519 \, g_\oplus$ that is nearly 5 times too large numerically.

Requirement (iii) in Press [1], that we are about as big as we can be without breaking when falling, led him to estimate that the energy released by a human of size L_H and mass $M_H \sim (m_p/a_0^3) L_H^3$, $E \sim M_H g L_H$, be of the order of the energy needed to break the human by disrupting a two-dimensional surface containing of the order of $(M_H/m_p)^{2/3}$ atoms, which is of the order of $(L_H/a_0)^2$. Taking the energy needed per atom to be ϵRy, this then is equivalent (modulo numerical factors of order unity that I am dropping) to saying that the weight of a proton, $m_p g$, be

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comparable to ϵ times the electrical force between two protons separated by the distance L_H, that is, to $\epsilon e^2/L_H^2$ in units with $4\pi\epsilon_0 = 1$.

Dropping the ϵ factor for the moment for simplicity, this last description of Press's criterion (not explicitly stated as such in Press [1] or in Press and Lightman [2]) may be explained as follows: If $N_c = L_H/a_0$ represents the number of atoms in a column one atom thick (each atom of mass approximated to be m_p, since we are ignoring factors of both the atomic mass number A and the nuclear charge number Z), then the Press criterion is that the energy released by this column of atoms falling a distance L_H, namely roughly $N_c m_p g L_H$, is comparable to the energy needed to break a chemical bond, which when one sets $\epsilon = 1$ is roughly a Rydberg or roughly the electrostatic potential energy of two protons separated by a distance a_0, namely e^2/a_0. Because of the $1/r$ falloff of the electrostatic potential, this is the same potential energy as that between one proton and a collection of N_c protons (ignoring the mutual potential energy between those N_c protons) at a distance N_c times greater than a_0, which is L_H. That energy in turn is the force between the one proton and the N_c protons at separation L_H, multiplied by this separation distance L_H, namely $N_c(e^2/L_H)^2 L_H$. Equating this to the energy of fall, roughly $N_c m_p g L_H$ by the Press criterion with $\epsilon = 1$, gives $m_p g \sim e^2/L_H^2$, which is the condition that the proton weight be comparable to the electrostatic force between two protons separated by L_H. When one reinserts the factor of ϵ or $\beta^{1/2}$, one gets that Press's criterion is that the proton weight should be about ϵ or $\beta^{1/2}$ times smaller than the electric force between two protons separated by a distance L_H given by the size of the organism.

If I now use the Press-Lightman [2] approximation (when their ε factor is dropped) that the Press $\epsilon \sim \beta^{1/2}$, then I get that Press's estimate for the height of man is

$$L_H \sim \beta^{1/8} m_e^{-1} m_p^{-1/2} e^{-3/2} = m_e^{-7/8} m_p^{-5/8} e^{-3/2} = \beta^{1/8}\gamma^{1/4} a_0 \approx 0.02181\,\text{m}. \qquad (13)$$

If one replaces β by Press's ϵ^2, then Press's expression [1] agrees with the first expression on the right hand side above, except that Press has a numerical factor of 2 that I have dropped. He writes the numerical value of L_H as $2.6(\epsilon/0.003)^{1/4}$ cm.

Press [1] recognizes that his estimated value is about 100 times smaller than the observed value and notes that he has underestimated man's breaking strength by a factor of about $10\,000$–$100\,000$ by assuming "implicitly that man was 'brittle,' i.e., that the energy of the fall would be concentrated as stress along his weakest fault plane. ... Probably the reason for this excess strength [over the estimate above] is that man's molecular structure is polymeric rather than amorphous, so that stresses are distributed over a rather larger volume than that of a single monatomic fault plane."

Press and Lightman [2] note that this size estimate that gives roughly 3 cm is the same as "the maximum size of water drops dripping off a ceiling." Barrow and Tipler [4] write, "The size estimates given by Press are a better estimate of the size of a creature able to support itself against gravity by the surface tension of water which is some fraction of the intermolecular binding energy, say $\epsilon\alpha^2 m_e$ per unit area, and Press's size limits, ~ 1 cm, more realistically correspond to the maximum dimension of pond-skaters rather than people."

2 Revised Estimates

Brandon Carter [3] has recently suggested that instead of Press's third requirement, one should think of a basic biological velocity $\tilde{v}$ such that $\tilde{m}\tilde{v}^2 \sim kT \sim \epsilon e^4 m_e$ with $\tilde{m} \sim \tilde{\epsilon}^{-1} m_p$ being "a mass scale characterising relevant large biochemical molecules such as proteins." Carter suggests that his new small numerical factor $\tilde{\epsilon}$ "might tentatively be taken to be given by $\tilde{\epsilon} \approx 10^{-3}$." With his formulation of $g \approx \epsilon^{1/2} e^5 m_e^2$, Carter suggests that the maximum height difference $\tilde{\ell}$ between different parts of the organism will be given by a formula that implies

$$\tilde{\ell} \sim \tilde{v}^2/g \sim \epsilon^{1/2}\tilde{\epsilon}e^{-1}m_e^{-1}m_p^{-1} = \epsilon^{1/2}\tilde{\epsilon}\gamma^{1/2}a_0 \approx 3\,200\,\mathrm{m}, \tag{14}$$

where I have used Press's value $\epsilon \sim 3 \times 10^{-3}$ and Carter's value $\tilde{\epsilon} \sim 10^{-3}$ to make the numerical evaluation given at the end.

This value is over $100\,000$ times the estimate of Press for the height of man and is unfortunately too large by a factor of about $1\,000$. If one said that $\epsilon \sim \beta^{1/2}$ and that $\tilde{\epsilon} \sim \beta$, just as I wrote Press's estimate as $L_H \sim \beta^{1/8}\gamma^{1/4}a_0 \approx 0.022$ m in terms of just β, γ, and the Bohr radius a_0, so I could do the same to express Carter's estimate as $\tilde{\ell} \sim \beta^{5/4}\gamma^{1/2}a_0 \approx 4\,900$ m.

It is helpful that Carter has found an argument that gives a higher power of γ, the huge ratio ($\sim 10^{36}$) between the electrical and gravitational forces between two protons, but unfortunately his argument seems to lead to too great a power. However, it motivated me to think of the following third argument that incorporates some of the reasoning of Press and Lightman [2] of how hard we can work and how fast we can run:

Press and Lightman [2] argue that the power of a large animal, say the horsepower, is "limited by cooling through the animal's surface area, and that resting metabolism is scaled to keep the resting animal tolerably warm." They give the estimate,

$$\text{(horsepower)} \sim \Delta T \times \text{(conductivity)} \times \text{(area)}/\text{(skin depth)}. \tag{15}$$

They go on to note, "If we had no knowledge of the observed parameters, we could use $\Delta T \approx T$, area of order h^2, skin depth of order h (where h is given by [Press's estimate]), and conductivity as given by (10)," which I can write first in their notation and then in Planck units (after dropping the factor of $1/2$ in the definition of the Rydberg) as

$$\text{(conductivity)} \sim (\mathrm{Ry}/a_0\hbar)(m_e/m_p)^{\frac{1}{2}}k \sim \alpha^3\beta^{1/2}m_e^2 = \alpha^4\beta^{5/2}\gamma^{-1}. \tag{16}$$

Then using $T \sim \alpha^2\beta^{1/2}m_e$ in Planck units from Eq. (8) gives the power in Planck units as

$$P \sim \text{(conductivity)}Th \sim \alpha^5\beta m_e^3 h, \tag{17}$$

in terms of the size h of the organism. However, here I shall not follow Press and Lightman [2] in using Press's estimate [1] for the height of man, $h \sim L_H$.

Press and Lightman go on to ask, "How fast can we run?": "We run in an extremely dissipative fashion. To run at velocity v, we must renew practically our whole kinetic energy every stride, that is, every motion through our body length h. The power needed to run at velocity v is therefore of order mv^3/h." Therefore, we shall set

$$P \sim mv^3/h. \tag{18}$$

Here m is the mass of the organism, which at the density of roughly m_p per cubic Bohr radius, just as assumed above for the earth, gives for an organism of volume h^3

$$m \sim (m_p/a_0^3)h^3 \sim \alpha^3 m_e^3 m_p h^3. \tag{19}$$

Now I shall follow Carter's [3] suggestion of $\tilde{\ell} \approx \tilde{v}^2/g$ and set $h \sim v^2/g$, though with slightly different motivation. Carter argues, "Assuming that such a velocity $\tilde{v}$ characterizes the relevant energies, pressures, and tensions (as involved for example in the pumping of blood) in an organism, it will provide an upper limit $g\ell \overset{<}{\sim} \tilde{v}^2$ on the supportable value of the gravitational energy per unit mass associated with a height difference ℓ between different parts of the body of the organism. This suggests that a biological land (though not necessarily sea) organism will be able to have a maximum size $\tilde{\ell}$ and a corresponding biological clock timescale $\tilde{\tau}$ given by

$$\tilde{\tau} \approx \frac{\tilde{\ell}}{\tilde{v}} \approx \frac{\tilde{v}}{g}.'' \tag{20}$$

Here I shall not follow Carter's suggestion for estimating the value of $\tilde{v}$ to use in deriving $\tilde{\ell}$ but shall use instead the Press-Lightman suggestion $P \sim mv^3/h$. Then the connection between my v and my h will not be Carter's idea about the pumping of blood (valid as that might be if one could get a reasonable $\tilde{v}$), but rather the idea that the (land) organism is assumed to be able to run and hence attain a velocity v that at least equals the speed of a pendulum of length h (now representing the leg length of the animal, which will be assumed to be of the order of h) of large-amplitude swinging, so $v \overset{>}{\sim} \sqrt{gh}$ and $P \overset{>}{\sim} mg^{3/2}h^{1/2}$.

To put it another way, it is assumed that a running animal has enough power to jump upward by an amount at least comparable to its height h. The energy for this is $E \sim mgh$, and the time t over which this energy must be exerted must be less than the time to fall a distance h, giving $t \overset{<}{\sim} \sqrt{h/g}$, so the power must be $P \sim E/t \overset{>}{\sim} mg^{3/2}h^{1/2}$.

For the tallest running animal (e.g., a giraffe, or a human under the approximation that both are large land animals of the same order of magnitude of size), let us say that these inequalities for running are saturated and that the resulting maximum value for h is henceforth called H (e.g., to avoid confusing it with the estimate Press and Lightman [2] quote for h from Press [1]):

$$v \sim \sqrt{gH}, \tag{21}$$

$$P \sim mg^{3/2}H^{1/2} \sim \alpha^3 m_e^3 m_p g^{3/2} H^{7/2}, \tag{22}$$

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where I have used Eq. (19) for the tallest running organism mass m in terms of its size $h = H$.

Now equating this estimate of the power necessary for running with the estimated limit on the power from Eq. (17) for the cooling rate, and also inserting the estimate of Eq. (12) for the acceleration of gravity g on a habitable planet from Press's requirements (i) and (ii) but not (iii), one gets that the following estimate for the height of a giraffe:

$$H \sim \alpha^{4/5} m_e^{2/5} m_p^{-4/5} g^{-3/5} \sim \alpha^{-7/10} m_e^{-19/20} m_p^{-13/20} \sim \beta^{1/20} \gamma^{3/10} a_0 \approx 2.44 \,\text{m}. \quad (23)$$

Unlike the previous estimates of both Press [1] and Carter [3], this estimate is within a factor of 2–3 of the height of the tallest running animal, the tallest land animal, the giraffe.

It is interesting that when one writes the result as the Bohr radius a_0 multiplied by the appropriate powers of α, β, and γ, the fine structure constant α drops out, and the multiple of the Bohr radius involves only the ratio of electron and proton masses, $\beta \equiv m_e/m_p$ and the ratio of the electrical to gravitational forces between two protons, $\gamma \equiv e^2/(Gm_p^2)$ (in units with $4\pi\epsilon_0 = 1$), with Planck's constant not appearing anywhere beyond its appearance in the Bohr radius $a_0 = 4\pi\epsilon_0 \hbar^2/(m_e e^2)$.

It is also interesting that the power of the mass ratio β is so small. Since $\beta^{1/20} \approx 0.6868$ is within a factor of 2 of unity, and since I have been cavalierly dropping many other factors of 2, one can drop this factor in Eq. (23) to obtain a simplified equation for the height of a giraffe that actually works even better empirically (though it is still not quite the maximum observed height of giraffes):

$$H \sim \gamma^{0.3} a_0 \approx 3.56 \,\text{m}. \quad (24)$$

That is, the height of a giraffe is here estimated to be roughly the Bohr radius multiplied by the 0.3 power of the ratio of the electrical and gravitational forces between two protons.

One can also use these estimates of the height of a giraffe to estimate that the total number of nucleons (or atoms, which will be of the same order, modulo the average atomic number of the molecules that is yet another number of order unity that I am ignoring) in a giraffe is either

$$N \sim \beta^{3/20} \gamma^{9/10} \approx 9.84 \times 10^{31}, \quad (25)$$

or, using the simplified Eq. (24) that drops the β factor,

$$N \sim \gamma^{0.9} \approx 3.04 \times 10^{32}. \quad (26)$$

The analogous masses are then

$$m \sim \beta^{3/20} \gamma^{9/10} m_p \approx 165\,000 \text{kg} \quad (27)$$

or

$$m \sim \gamma^{0.9} m_p \approx 508\,000 \text{kg}. \quad (28)$$

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The first of these mass estimates, from the slightly more sophisticated estimate of the height H, is reasonably good for blue whales, but it is not supposed to apply to sea creatures but only to running land creatures. If one goes to elephants as the most massive running land animal alive today, running up to $12\,000$ kg, the mass estimates above are a bit too high, from about 14 to about 42 times the largest recorded mass of a land animal species alive today. However, it is gratifying that even the mass estimates are within two orders of magnitude of observed values.

From the first of my giraffe height estimates, Eq. (23), one can also obtain estimates for the time to take a stride, the stride time

$$t \sim \sqrt{H/g} \sim \alpha^{2/5}m_e^{1/5}m_p^{-2/5}g^{-4/5} \sim \alpha^{-8/5}m_e^{-8/5}m_p^{-1/5} \sim \alpha^{-1}\beta^{-3/5}\gamma^{2/5}a_0/c \approx 0.601\mathrm{s}. \tag{29}$$

Carter [3] has noticed that the characteristic time of our mental and other biological processes is in Planck units of the order of $m_e^{-1}m_p^{-1} \approx 0.0168$ seconds, which coincidentally is only 0.57% larger than the period of the 60-Hertz alternating current used in North America, since $m_e m_p \approx 59.66$ Hz. One may express the stride time above in terms of Carter's simple characteristic time as

$$t \sim \alpha^{-3/2}\beta^{-3/5}\gamma^{-1/10}m_e^{-1}m_p^{-1} \approx 35.8\,m_e^{-1}m_p^{-1}. \tag{30}$$

The stride time thus has a slightly less positive power of the gravitational coupling (in γ^{-1}) than Carter's characteristic time (which by itself would tend to make the stride time shorter than Carter's time), but the difference in the powers of the fine structure constant α and the electron-proton mass ratio β gives a large factor, nearly 1.46×10^5, whose logarithm is about 1.43 times larger than the negative of the logarithm of the $\gamma^{-1/10} \approx 0.000\,246$, so in the end the stride time estimate comes out nearly 40 times larger than Carter's electrifyingly simple characteristic time.

One can similarly calculate the running velocity

$$\begin{aligned} v \;\sim\;\; & H/t \sim \sqrt{Hg} \sim \alpha^{2/5}m_e^{1/5}m_p^{-2/5}g^{1/5} \sim \alpha^{-9/10}m_e^{-13/20}m_p^{-9/20} \\ \sim\;\; & \alpha\beta^{13/20}\gamma^{-1/10}\mathrm{c} \approx 1.36 \times 10^{-8}\mathrm{c} \approx 4.07\,\mathrm{m/s} \approx 14.6\,\mathrm{km/hr} \approx 9.10\,\mathrm{mph}. \end{aligned} \tag{31}$$

This purely theoretically derived estimate for the fastest animal running velocity is perhaps roughly the speed that a theorist like me can run, but it is slightly less than 40% of the average speed of 10.35 m/s or 37.27 km/hr or 23.16 mph (miles per hour) of Michael Johnson in running 200 meters in 19.32 seconds, and it is less than 20% of the maximum speed of a cheetah.

It is interesting that if one compares this estimate for the top running speed of an animal with the remarkably good estimate of Eq. (11) for the speed of a low satellite, one finds that

$$v \sim (\beta\gamma)^{-1/10}\,v_{\mathrm{satellite}} = \beta^{-1/5}\left(\frac{Gm_e m_p}{e^2}\right)^{0.1}v_{\mathrm{satellite}}, \tag{32}$$

so the speed of a running animal is estimated to be just a few times larger (by a factor of $\beta^{-1/5} \approx 4.5$) than the speed of a low satellite multiplied by the one-tenth power of the ratio of the gravitational to the electrical attractions between the

electron and the proton in a hydrogen atom. It can perhaps give one a bit of a feel for this tiny ratio of forces if one realizes that it is even millions of times smaller (by a factor of $\beta^2 \approx 0.3 \times 10^{-6}$) than the tenth power of the ratio of the speed that one can run to the speed of a satellite.

3 Using Anthropic Estimates for the Charge and Mass of the Electron and Proton

So far I have used the observed values of the charge and mass of the electron and proton to give new estimates for the height, stride time, and running velocity of the giraffe, the tallest running animal. However, one can also derive anthropic estimates for all of these quantities [5] that do not depend upon *any* measured continuous parameters or coupling constants, so it may be of interest to insert these purely mathematical values into the estimates above.

Basically, [5] uses the anthropic results of [6, 7, 8] and the renormalization group formulas of [9] to derive, using no fudge factors at all,

$$\alpha \ln \alpha \sim -\frac{\pi}{100}, \tag{33}$$

$$\beta \sim \alpha^2, \tag{34}$$

$$\gamma \sim \alpha^{-19}, \tag{35}$$

$$m_e \sim \alpha^{12}, \tag{36}$$

$$m_p \sim \alpha^{10}, \tag{37}$$

$$a_0 \sim \alpha^{-13}, \tag{38}$$

$$M_s = L_s \sim \alpha^{-20}, \tag{39}$$

Let us take the solution of Eq. (33), with an equal sign rather than the $\sim$ sign, as α_a (with subscript a for "anthropic"), with the numerical solution

$$\alpha_a \approx 0.006\,175\,533\,381. \tag{40}$$

This then gives

$$\alpha \sim \alpha_a \approx 0.846\alpha, \tag{41}$$

$$1/\alpha \sim 1/\alpha_a \approx 162 \approx 1.18/\alpha, \tag{42}$$

$$\beta \sim \alpha_a^2 \approx 0.000\,038 \approx 0.070\,\beta, \tag{43}$$

$$\gamma \sim \alpha_a^{-19} \approx 9.5 \times 10^{41} \approx 770\,000\,\gamma, \tag{44}$$

Then my estimate for the giraffe height would be

$$H \sim \alpha_a^{-5.6} a_0 \approx 2.4 \times 10^{12} a_0 \approx 125 \,\text{m}, \tag{45}$$

where for the last number I am assuming that if α, β, and γ were changed to the 'anthropic' values given here, then the meter would be defined so that it still equaled $1.89 \times 10^{10} a_0$. This would then imply that the purely theoretical anthropic estimate for the number of atoms in the largest land animal in a typical biophilic region of the universe, multiverse, or holocosm would be of the order of

$$N \sim \alpha_a^{-16.8} \sim 10^{37}. \tag{46}$$

A number of this rough order of magnitude might be a crude estimate for the maximally complex living being in the holocosm. It seems plausible that there might be significantly more complex beings that that, but also that their numbers might be tailing off sufficiently rapidly that this number gives a rough upper limit.

Acknowledgments

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